SCATTERING OF NEUTRONS BY PROTONS AT HIGH ENERGIES

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ABSTRACT. It has been shown here that the recent experimental results on the neutron-proton scattering at 260 Mev energy favours the Möller-Rosenfeld interaction and rules out the pseudoscalar theory. Contrary to the general expectation that the Born approximation is valid before the influence of the radiation reactions needs to be taken into account, it is found that the Born apporoximation, which is reasonably valid for 260 Mev energy, gives the theoretical value twice as large as the experimental one but the inclusion of the radiation damping term brings the theoretical value in good agreement with the experimental result.

INTRODUCTION

In the theoretical investigation of the scattering of neutrons by protons, one is faced with the following difficulties : The exact nature of the meson field giving rise to the nuclear interaction is not definitely known. All the four representatives of the meson field so far proposed give rise to nuclear interactions containing inadmissible singularity at $r \rightarrow 0$, the presence of which prevents us from having proper solution of the Schrödinger equation. However, without knowing the proper solution, it is possible to calculate the neutron-proton scattering cross-section in the Born approximation which can be applied only in the high energy region. On the other hand, for such relativistic energies, the influence of the field reactions, namely the radiation damping, cannot be neglected, the integral equation describing the influence of the radiation damping is mathematically very complicated.

In the earlier paper the present author investigated the cross section of neutron-proton (in short n-p.) scattering at high energy region in the Born approximation, the Möller-Rosenfeld (in short M.-R.) version of the meson field was chosen for the interaction between the neutron and the proton, the effect of the retardation is accounted for by the mechanism of exchange of charged and neutral mesons of vector and pseudoscalar type. Using the ordinary Born approximation, the cross section for the n-p seattering is found to increase indefinitely with increasing energy of the incident neutron in the relativistic region, no matter whether the vector or the pseudoscalar or the M.-R. mixed meson field is used. In a subsequent paper (Basu, 1950) it was shown that the influence of radiation damping on n-p scattering cross-section is sufficient to prevent the indefinite increase of the cross-section, it was found that the inclusion of radiation damping makes the cross-section decrease as $1/E'$ for high values of E', where E' is the energy of the neutron in the rest system. At that time the experimental data on n-p scattering at high energy region was not available to make comparison possible with our theoretical findings.

It is our object in the present paper to compare the theoretical findings of our previous two papers with the recent experimental results of n-p scattering at 260 Mev energy which have appeared in three papers by Kelly, Leith, Segre and Wiegand (1950) ; Fox, Leith, Wouters and Mackenzie (1950) and Dejuren (1950). Such a comparison would give us an indication as to what will be our choice of the meson field. We shall see here that the M.-R. version of the meson field gives a good fit of the theoretical findings with the experimental values of the total cross section of the n-p scattering at 260 Mev energy.

The M.-R. interaction in the non-relativistic limit contains three constants, one is related to the mass of the meson and the other two represent the strength of the charge and spin coupling. The latter two can be fixed from a comparision of the calculated and the experimental values of the binding energies of the triplet and singlet states of deuteron. The mass of the meson, which determines the first constant, is known from measurements of cosmic ray and artificially produced mesons. It is the π-meson which is responsible for the nuclear force and its mass is 286 electron masses (m_e) (Bishop, 1949). In the M.-R. theory, the meson mass corresponding to the pseudoscalar part of the field is assumed to have the same value as the voctor counterpart, the pseudoscalar mesons would be spinless particles; there is no experimental evidence yet of spinless mesons of the same mass as 286 m_e which enter into the M.-R. combination. The μ-meson, whose mass and spin are 215 (Ratallack and Brode, 1949) and $\frac{1}{2}$ respectively, has very little influence on the nuclear forces ; on the other hand, the contribution to the nuclear force of the τ-mesons of mass 800-1000 m_e has not been incorporated in the theory. The three constants of the M.-R. interaction having been thus fixed from the experimental values of the deuteron, binding energies in the singlet and triplet states and direct mass measurements of the observed mesons, a check on the correctness of the M.-R. interaction can then be made by comparing the theoretical value of the n-p scattering cross section so calculated with the recently published experimental values on the same. It is shown here that the value of the total cross section of n-p scattering at 260 Mev energy favours the M.-R. interaction and further, under certain approximations, the influence of radiation damping at this energy is to reduce the cross-section by about one half.

The M.-R. interaction is preferable to other meson field forces because of two other evidences coming out of the experimental results on n-p scattering at different energies. The value of the differential cross-section of n-p scattering at 40, 90 and 260 Mev energies (Kelly and others, 1950) shows a minimum at 90° angle and an increase for forward and backward angles. The results for backward scattering are subject to considerable errors and the values for angles smaller than 40° are not available yet. In view of the fact that complete and very reliable results of the differential cross-section of n-p scattering are not available yet, it is not possible to make any definite conclusion regarding the charge symmetry of the M.-R. theory.* If we draw curves through the experimental points of Kelly and others, we can find at 40, 90 and 260 Mev energies the anisotropy of scattering which is expressed as the ratio $T=dq(\pi)/dq(\pi/2)$. Moreover, the areas under the curves would give us relative measures of the total cross-sections for the three energy values. We shall see that T increases and Q decreases with increasing energy values of the scattered neutrons. Hulthén (1944) has investigated the anisotropy ratio for different meson fields in the non-relativistic limit and he finds that in the M.-R. theory the ratio tends towards infinity for increasing energy values which is borne out by the experimental finding. According to Hulthén, the anisotropy ratio for the symmetrical pseudoscalar theory decreases with increasing energy and tends to the limit 1 for very high energies, the same ratio for the symmetrical vector meson theory tends to the limit $3+2g_1^4/g_2^4$ (≈ 3.1) for every high energies, whereas, the experimental value for 260 Mev neutron energy is 9. The value of the total cross-section for 260 Mev neutron energy is nearly one half the value for 90 Mev. We shall presently see that this decrease of the total cross-section with increasing energy values can be satisfactorily explained in the M.-R. theory but not in the pseudoscalar theory.

There is, however, one difficulty with the M.-R. theory in which the tensor force is eliminated in the non-relativistic limit, as such the M.-R. theory fails to explain the presence of the quadrupole moment of the deuteron. The tensor force, which contains the inadmissible singularity at $r\to 0$ of the type $1/r^3$ but explains the presence of the quadrupole moment, appears in both the vector and pseudoscalar theory. However, the quadrupole moment, as calculated from the pseudoscalar theory, agrees in sign but not in magnitude with the experimental value. In the non-relativistic limit the tensor force term of the pseudoscalar theory has the same form but with the opposite sign as that of the vector theory ; that is why the mixed M.-R. field eliminates the tensor force in the static interaction if the constants are given the same value for both the fields. The subsequent generalzation of Schwinger (1942). does not give the correct value of the quadrupole moment of the deuteron as has been shown by Jauch and Hu (1944) and Ramsey (1948). It may be mentioned in this connection that the elimination of the tensor force in the M.-R. theory is effected only in the

* From the angular distribution of the scattered neutrons it was tentatively suggested by Serber (quoted by Yukawa, 1949) that a mixture of the exchange and non-exchange forces in the ratio 1 : 1 gives a good fitting with the experimental findings. Prof. Rosenfeld thinks (in course of a conversation) that the experimental results that are in progress may not bear out Serber's suggestion.

static potential, the tensor force reappears, as has been shown by Hu (1945), in the relativistic region.

The Born approximation, which we have applied in our calculations, is valid when the energy of the colliding particles is large compared with their interaction energy. From calculations regarding the criterion of the validity of the Born approximation, it appears that it is valid for energies above $50-100$ Mev (measured in the centre of mass system). While on the high energy side, when the Born approximation becomes valid, the question arises as to the necessity of including the influence of radiation damping. Heitler (1948) and Janossy and McConnell (1950) have probed into the problem of finding the energy region within which the Born approximation is sufficiently good and yet the influence of the radiation damping is not pronounced. According to Heitler (1948), the Born approximation becomes valid before the effects of the field reactions have to be taken into account; however, we shall see that a closer examination makes this view untenable. The Born approximation is reasonably valid for 63.0 Mev energy (this value in CM corresponds to 260 Mev energy in the rest system), and for this energy we find that the theoretical value of the cross-section without the inclusion of radiation damping is twice the experimental value; further if the influence of the radiation damping is included, there is reasonable agreement between the theoretical and the experimental values. It means that when the Born approximation is applicable, it becomes also necessary to include the influence of the radiation reactions.

THE MÖLLER-ROSENFELD INTERACTION

The M.-R. interaction between two nucleons in the non-relativistic limit takes the simple form given by

$$W^{12} = (\boldsymbol{\tau}^1\boldsymbol{\tau}^2)\left\{ g_1{}^2 + g_2{}^2(\boldsymbol{\sigma}^1\boldsymbol{\sigma}^2) \right\}\frac{e^{-\chi r}}{r} \qquad \ldots \quad (1)$$

where $\boldsymbol{\tau}^1,\boldsymbol{\tau}^2$; $\boldsymbol{\sigma}^1$, $\boldsymbol{\sigma}^2$ denote the isotopic spin-vectors and spin operators of the two particles numbered 1 and 2 and r is the distance between the two nucleons. The constant $\chi=\mu/\hbar c$, μ being the mass of the meson in energy units and $\hbar$, c have their usual meanings. The constants g_1 and g_2, which have the dimension of electric charge, denote the strengths of the two types of coupling.

The Schrödinger equation of the deuteron with the above potential can be solved numerically, then the values of the binding energies in the triplet and singlet states make it possible to determine g_1 and g_2. The recent calculation of Fröhlich, Huang and Sneddon (1947) gives their values as

$$\frac{g_1{}^2}{\hbar c} = 0.0246, \quad \frac{g_2{}^2}{\hbar c} = 0.000306\,\frac{m_v}{m_e} + 0.0085 \qquad (2)$$

The experimental value for the ratio of the meson mass to that of the electron (m_u/m_e) is 286, in which case the constants become

$$\frac{g_1^2}{\hbar c} = 0.024, \quad \frac{g_2^2}{\hbar c} = 0.096 \qquad \cdots \quad (3)$$

The constant g_1^2 is smaller than g_2^2 by a factor 4, the scattering cross-sections are proportional to the fourth power of g_1 and g_2. We shall see in the next section that the scattering at high energy due to the g_1 part of the interaction can be neglected because of two reasons : (1) g_1^4 is much smaller than g_2^4 and (2) the scattering cross section due to the g_1 interaction falls quite sharply with increasing energy of the incident particle.

N-P SCATTERING CROSS SECTION

The exact relativistic expression for neutron-proton interaction according to the M.-R. version of the meson field is given in equations (5) and (6) of the earlier paper (Basu, 1949). This interaction includes in it such terms as would make it free, in the non-relativistic limit, from contact potentials of the form of δ-function. Such an elimination of the δ-functions has an important bearing on the g_1-scattering of neutrons by protons, the cross section of which decreases with increasing energy and tends to a constant in the extreme relativistic region. This decrease of cross section with increasing energy is reversed if the δ-functions are not eliminated in the above mentioned way.

The calculation of the scattering cross-section with the M.-R. interaction is very laborious, the part played by the g_1-interaction will be shown to be small enough to be neglected, as such, we have avoided some labour by calculating the relative contributions of the vector and the pseudoscalar meson fields separately, we shall see that so far as g_1-scattering is concerned the contribution of the pseudoscalar field is exceedingly small compared with that of the vector field. Representing the nucleon by the Dirac spinor, the total cross section of the n-p scattering due to the g_1-interaction of the vector (charged or neutral) meson field is given by

$$Q_0{}^v = \frac{2\pi}{\chi^2} \frac{g_1^4}{\hbar^2 c^2} \frac{a^2}{1+x^2} \left[\frac{2}{a^4} \left\{ 1 + \frac{1}{1+4a^2x^2}(2a^2x^2 + 16a^4x^4) - \left(1 + \frac{1}{4a^2x^2}\right) \log(1+4a^2x^2) \right\} + \frac{8}{a^2} \left\{ 1 - \frac{1}{4a^2x^2} \log(1+4a^2x^2) \right\} + \frac{8}{1+4a^2x^2} \right] \qquad \cdots \quad (4)$$

where a is the ratio of the nucleon mass to the meson mass and is 6.42 for the meson mass being equal to 286 m_e, x is the ratio p_0/M, p_0 and M both expressed in energy units are the momentum in the centre of mass system and the rest mass of the incident neutron or proton. The reciprocal of χ has the dimension of length and its value for the meson mass 286 m_e

is 1.35×10^{-13} cm. The numerical value of the above expression is $Q_0^{v}(g_1) = 0.11 \times 10^{-26}$ cm^2 for 260 Mev neutron energy.

The similar cross-section due to the g_1-interaction of the pseudoscalar (charged or neutral) meson field has the form

$$Q_0^{ps} = \frac{2\pi}{X^2}\frac{g_1^4}{\hbar^2 c^2}\frac{a^2}{1+x^2}\left|\frac{1}{a^4}\left\{\frac{1}{2} + \frac{1}{2(1+4a^2x^2)} - \frac{1}{4a^2x^2}\log(1+4a^2x^2)\right.\right. \qquad (5)$$

$$Q_0^{ps}(g_1) = 0.00005 \times 10^{-26}\text{cm}^2 \text{ for } 260 \text{ Mev neutron energy.}$$

It is evident that in a mixture of the vector and pseudoscalar fields, the contribution to the g_1-scattering at 260 Mev energy, of the pseudoscalar part of the field is smaller than the vector part by a factor 5×10^{-4}; as such in the g_1-scattering only the addition of the pseudoscalar field to the vector field does not give anything different from what would be obtained from the pure vector field. However, if we combine the charged and the neutral field in a way to ensure charge symmetry, the value of the cross section would be roughly multiplied by a factor 2. Hence the value of the total cross-section due to the g_1-interaction of the M.-R. theory will be approximately

$$Q_0^{MR}(g_1) \approx 0.22 \times 10^{-26} \text{ cm}^2 \text{ for } 260 \text{ Mev neutron energy.}$$

We shall see in the next paragraph that the total scattering cross-section due to the g_2 interaction alone of the M.-R. theory has the value 7.02×10^{-26} cm^2 (for 260 Mev neutron energy) which is roughly 30 times the value of the above cross section due to the g_1-interaction at the same energy value. Since the evaluation of the general expression for the cross section of the n-p scattering is exceedingly complicated when both g_1 and g_2 of the relativistic M.-R. interaction are different from zero; we shall simplify our calculations by putting $g_1 = 0$ it would mean that the value of the cross section so obtained would be less than that of the exact expression which includes both g_1 and g_2 by 5 to 10 per cent.

If we neglect the contribution of the g_1-scattering for reasons stated above, the total cross-section of n-p scattering with the M.-R. version as the interaction between the neutron and proton is given by

$$\begin{aligned} Q_0^{MR} = \frac{2\pi}{X^2}\frac{g_2^4}{\hbar^2 c^2}\frac{a^2}{1+x^2}\Bigg[& x^4\Bigg\{\frac{32}{3} + \frac{6}{a^2x^2} + \frac{25}{24a^4x^4} + \frac{5}{8a^4x^4(1+4a^2x^2)} \\ & - \left(\frac{13}{3a^2x^2} + \frac{41}{24a^4x^4} + \frac{5}{12a^6x^6} + \frac{5}{4a^2x^2(1+2a^2x^2)}\right)\log(1+4a^2x^2)\Bigg\} \\ & + 4x^2\left\{\frac{1}{a^2x^2} + \left(\frac{19}{12a^2x^2} - \frac{1}{4a^4x^4} - \frac{5}{4a^2x^2(1+2a^2x^2)}\right)\log(1+4a^2x^2)\right\} \\ & + 3\left\{\frac{10}{1+4a^2x^2} - \frac{1}{a^2x^2(1+2a^2x^2)}\log(1+4a^2x^2)\right\}\Bigg] \qquad \ldots (6) \end{aligned}$$

The above expression (6) for Q decreases with increasing x in the low energy region from $x=0$ to about $x=0.5$ (the latter value would correspond to a kinetic energy of the value 110 Mev in the centre of mass system or 470 Mev in the system in which the proton is at rest) after which it increases in the relativistic region indefinitely with increasing values of x. The value of Q is sensitive to the value of the meson mass. The numerical values of the cross sections as calculated from the above expression are as follows :

$$Q_0^{MR} = \begin{cases} 7.02 \times 10^{-26} \text{ cm}^2 & \text{for 260 Mev neutron energy} \\ 12.73 \times 10^{-26} \text{ cm}^2 & \text{,, 90 Mev ,, ,,} \end{cases}$$

It must be borne in mind that the relativistic Born approximation which we have employed in our calculations is not quite good for 90 Mev neutron energy which would correspond to 21.6 Mev energy in the centre of mass system.

To compare the relative contributions of the vector and the pseudoscalar fields in the mixed field of the M.-R. theory, we consider the g_2-scattering due to the pseudoscalar interaction alone. Contrary to what we have seen for the g_1-scattering, we notice that the g_2-scattering due to the pseudoscalar interaction is larger than the same due to the M.-R. interaction. Bearing in mind that every interaction between two nucleons, in the non-relativistic limit, should be free from δ-functions, the total cross section for n-p scattering due to the pseudoscalar (charged or neutral) field is given by

$$Q_0^{ps} = \frac{2\pi}{\chi^2} \frac{g_2^4}{\hbar^2 c^2} \frac{a^2}{1+x^2} \frac{32}{9} \left[\frac{2}{3}x^4 + x^2 + \frac{3}{2} + \frac{9}{4(1+4a^2x^2)} - \frac{3}{2} \frac{1}{2a^2x^2} \log(1+4a^2x^2) \right] \quad \dots (7)$$

The cross-section Q_0^{ps} (7) increases indefinitely with increasing x right from $x=0$. The numerical values as calculated from the above expression are as follows :

$$Q_0^{PS}(g_2) = \begin{cases} 18.02 \times 10^{-26} \text{cm}^2 & \text{for 260 Mev neutron energy,} \\ 14.42 \times 10^{-26} \text{cm}^2 & \text{,, 90 Mev ,, ,,} \end{cases}$$

We notice that the g_2-scattering cross section, when calculated with the pseudoscalar interaction, is very much larger than the similar expression calculated with the M.-R. interaction.

If we denote the matrix elements of the process of scattering by H_{fi} the square of which is proportinal to the cross section, it is clear from a comparision of the numerical values of the cross-section, as calculated with the M.-R. interaction, and the interaction due to the pseudoscalar field that the absolute value of the matrix elements for the M.-R. interaction is less than the same for the pseudoscalar interaction round about 260 Mev energy.

It follows from the method of Hsüeh and Ma (1945) that the scattering cross section is influenced by the radiation damping in the following way :

$$Q = \frac{Q_o}{1 + b^2/a^2} \qquad \ldots \quad (8)$$

where Q_o denotes the ordinary cross section in which radiation damping has been neglected. b and a are given by

$$a = \sum_i \sum_f \int H_{if} H_{fi} \rho_f d\Omega_f \qquad (9a)$$

$$b = \pi \sum_i \sum_f \sum_{f'} \int\int H_{if} H_{ff'} H_{f'i} \rho_f \rho_{f'} d\Omega_f d\Omega_{f'} \quad . \qquad (9b)$$

The suffices f and i stand for the final and initial states, $\rho_{f'}$ denotes the density of energy levels corresponding to the state f'. The evaluation of a and b with the $H_{fs}^{\mathrm{M.-R.}}$ for the M.-R. interaction is the proper procedure, but such a procedure is exceedingly involved to calculate To simplify labour a and b have been calculated in a previous paper (Basu, 1950) with the matrix elements H_{fi}^{ps} for the pseudoscalar field. As $|H_{fi}^{\mathrm{M.-R}}|$ is smaller than $|H_{fi}^{ps}|$, therefore the exact value of $|b/a|$, as evaluated with the matrix elements $H_{fi}^{\mathrm{M\,R}}$ for the M.-R. interaction, would be somewhat smaller than what is calculated here with the pseudoscalar interaction. We give below the expression for the ratio b/a

$$\frac{b}{a} = \frac{g_2^2}{4} \frac{a^2 x}{(1+x^2)^{\frac{1}{2}}} \frac{F_b(x)}{F_a(x)} \qquad \ldots \quad (01)$$

where $F_a(x)$ and $F_b(x)$ are functions of the variable x and their forms are quoted for convenience in the Appendix. The numerical values are as follows :

$$1 + \frac{b^2}{a^2} = \begin{cases} 2.19 & \text{for 260 Mev neutron energy,} \\ 1.23 & \text{,, 90 Mev ,, ,, .} \end{cases}$$

It shows that the influence of radiation damping at 260 Mev energy is to reduce the ordinary cross section by about one half, which is by no means negligible. We regard the influence of damping as a correction factor to the ordinary expression for the scattering cross section, the procedure of calculating the damping term with the pseudoscalar interaction is justified on the ground that it gives the order of the correction. However, it must be remembered that the exact numerical values for the M.-R. interaction would be somewhat smaller than the above figures.

We now give below in a tabular form the numerical values so far derived from the theoretical expressions :

E'_{kin} (RS)	E_{kin} (CM)	x	$Q_0^{MR} \times 10^{26}$ cm^2.	$Q^{MR} \times 10^{26}$ cm^2. (with damping)	$Q^{ps} \times 10^{26}$ cm.	$Q^{ps} \times 10^{26}$ cm^2. (with damping)
90 Mev	21.6 Mev	0.219	12.7*	10.3*	14.4*	11.7*
260 Mev	63.0 Mev	0.372	7 0	3.2	18.0	8.2

E'_{Kin} denotes the kinetic energy in the system in which the protion is at rest whereas, E_{kin} is the corresponding kinetic energy in the centre of mass-at-rest-system.

We then compare the above with the experimental findings of different workers. The curves drawn through the experimental points of Kelly and others give us the following results.

Experimental Results of Kelly and Others.

E'_{kin} (RS)	$T = \frac{dQ(\pi)}{dQ(\pi/2)}$	Q (in arbitrary units)
40 Mev	1.6	25.1
90 Mev	3 7	13 4
260 Mev	9 2	7 0

The value of the total n-p scattering cross section has been measured by Fox, Leith, Wouters and MacKenzie (1950) who have used a double coincidence anthracene scintillation counter telescope as the neutron detector. The same measurement has been done by DeJuren (1950) using bismuth fission ionization chamber as the neutron detector.

E'_{kin} (RS)	Total cross section $Q \times 10^{26}$cm^2	
	Fox and others	DeJunren
85 Mev	8.3 ± 0.4	
95 ,,		7.3
270 ,,		3.8 ± 0.2
280 ,,	3.3 ± 0.3	

$$\frac{Q\ (260\ \text{Mev})}{Q\ (\ 90\ \text{Mev})} = 0.52 \qquad \text{(from Kelly and others)}$$

* We have marked the values of the cross section for 21·6 Mev energy (CM) with asterisks to indicate that the Born approximation is not good for such an energy.

$$\frac{Q\,(280\text{ Mev})}{Q\,(85\text{ Mev})}=0.40 \qquad \text{(Fox and others)}$$

$$\frac{Q\,(270\text{ Mev})}{Q\,(95\text{ Mev})}=0.52\pm0.03 \quad \text{(DeJuren)}$$

The theoretical value of the total scattering cross section after being corrected for radiation damping is 3.2×10^{-26} cm^2 and the two experimental values are $3.3\pm0.2\times10^{-26}$ cm^2 and $3.8\pm0.2\times10^{-26}$ cm^2. The agreement appears to be very good, the fact that our theoretical value is on the side of being slightly less than the experimental one is expected because the calculations can be improved in two ways each of which would increase the value slightly and as such the two together would make the agreement much better. The two improvements consist of taking both g_1-and g_2-interactions and the evaluation of the radiation correction with the M.-R. theory. The M.-R, theory allows us to fix the two constants from the deuteron problem, the determination of the third constant from the observed mass of the π-meson leaves no arbitrariness about the n-p scattering cross-section, as such the agreement of the theoretical and the experimental values of the scattering cross section shows that the deuteron and n-p scattering problem can be consistently framed in one picture with the help of the M.-R-theory.

It is quite evident, as was expected to be so, that the Born approximation is not good for 21·6 Mev energy (corresponding to 90 Mev in RS), for such low energies the Born approximation gives too large values for the cross section compared with what they should be. The pseudoscalar field alone fails to explain the n-p scattering results.

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APPENDIX

$$F_a(x) = \frac{8}{9}\left\{\frac{2}{3}x^4 + x^2 + \frac{3}{2} + \frac{9}{4}\,\frac{1}{1+4a^2x^2} - \frac{3}{4a^2x^2}\log(1+4a^2x^2)\right\}$$

$$F_b(x) = \frac{32}{9}\left\{\frac{16}{27}x^6 + \frac{4}{3}x^4 + 2x^2 + 1\right\} + \frac{1}{8}(x^2+1)\left\{x^4+4x^2+8\right\}A$$

$$-\frac{2}{3}(x^2+1)\left\{x^4+4x^2+7\right\}B - \frac{x^4}{18}(x^2+1)C + \frac{x^2}{32}\left\{2x^4+2x^2+7\right\}C^2$$

where (putting $\alpha = \frac{1}{2a^2x^2}+1$)

$$A = 16\alpha - 6\alpha^2 + (8 + 2\alpha - 16\alpha^2 + 6\alpha^3)\log\frac{\alpha+1}{\alpha-1}$$

$$+\left(\frac{5}{2} - 4\alpha - \alpha^2 + 4\alpha^3 - \frac{3}{2}\alpha^4\right)\left(\log\frac{\alpha+1}{\alpha-1}\right)^2$$

$$B = 2 - (\alpha - 1)\log\frac{\alpha+1}{\alpha-1}$$

$$C = 2\alpha - (\alpha^2 - 1)\log\frac{\alpha+1}{\alpha-1}$$

REFERENCES

Basu., 1949, *Proc. Roy. Ir. Acad.*, **52**, 127.
1950, *ibid* **53**, 31.
Bishop, 1949, *Phys. Rev.*, **15**, 1468.
DeJuren, 1950, *Phys. Rev.*, **80**, 27.
Fox, Leith, Wouters and MacKenzie, 1950, *Phys. Rev.*, **80**, 23.
Fröhlich, Huang and Sneddon., 1947, *Proc. Roy. Soc.*, **191A** 61.
Heitler, 1948, *Proc, Roy. Ir. Acad.*, **52**, 95,
Cf. Acta Phys. Austriaca Bd. **1**, 110.
Hsüeh and Ma., 1945, *Phys. Rev*,, **67**, 303.
Hu, 1945, *Phys. Rev.*, **67**, 339.
Hulthén, 1944, *Arkiv. Mat. Astr. Fys.* **31A**, No. 15.
Janossy and McConnell, 1950, *Proc. Roy. Ir. Acad.*, **53**.
Jauch and Hu, 1944, *Phys. Rev.*, **65**, 289.
Kelly, Leith, Segre, and Wiegand, 1950, *Phys. Rev.*, **79**, 96.
Ramsey, 1948, *Proc. Phys. Soc.*, **61**, 297.
Ratallack and Brode, 1949, *Phys. Rev.*, **75**, 1716.
Schwinger, 1942, *Phys. Rev*,, **61**, 287A.
Yukawa, 1949, *Rev. Mod. Phys.*, **21**, 474.